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Comments on "Convection–diffusion of solutes in media with piecewise constant transport properties" by Vaidya

Vaidya, D.S., Nitsche, J.M., Diamond, S.L. and Kofke, D.A., 1996, published a paper in Chem. Engng. Sci. 51, 5299–5312.

The aim of this letter is to point out that the authors overlooked a subtle aspect of the solution in Appendix B. Consequently, the solution given in Eq. (B13) and Eq. (B14) of the paper is incomplete.

The solution presented in the paper is

$$c^{\mathrm{I}}(\beta, \sigma) = f^{\mathrm{I}}(\beta), \tag{B13}$$

$$c^{\mathrm{II}}(\beta, \sigma) = [v_r^{\mathrm{I}}(\sigma + \alpha)/v_r^{\mathrm{II}}(\sigma + \alpha)] f^{\mathrm{I}}(\beta_2). \tag{B14}$$

This solution is not complete. For instance, we see that $c^{\mathrm{II}}(\beta, \sigma)$ does not satisfy the initial condition:

$$c^{\mathrm{II}}(\beta, 0) = 0. \tag{B11}$$

Although $v_r^{\mathrm{I}}(\sigma)$, $v_r^{\mathrm{II}}(\sigma)$ and $f^{\mathrm{I}}(\beta)$ are some smooth functions, the solution $c^{\mathrm{II}}(\beta, \sigma)$ in general is called the weak solution because $c^{\mathrm{II}}(\beta, \sigma)$ has a jump discontinuity at a characteristic curve. In the paper, the authors only paid attention to the discontinuity on the interface $\beta = L_{\mathrm{f}}(0)$, but ignored a jump at a characteristic curve.

For Eq. (B9), the characteristics are given by

$$\beta = \beta_0 + \int_0^\sigma v_r^{\mathrm{II}}(s)\,\mathrm{d}s.$$

Along the curves, $c^{\mathrm{II}}(\beta, \sigma)$ is constant. There exists a unique curve passing through the given point (β, σ) in the region

$$D = \{(\beta, \sigma) \,|\, \beta \geqslant L_f(0), \sigma \geqslant 0\}.$$

When $\beta = L_f(0)$ and $\sigma = 0$, we have $\beta_0 = L_f(0)$. Therefore, the region D is divided into two parts D_1 and D_2 by the curve l

$$\beta = L_f(0) + \int_0^\sigma v_r^{\mathrm{II}}(s)\,\mathrm{d}s,$$

which emanates from point $(L_f(0), 0)$ as shown in Fig. 1, where

$$D_1 = \left\{(\beta, \sigma) \,|\, \beta \geqslant L_f(0) + \int_0^\sigma v_r^{\mathrm{II}}(s)\,\mathrm{d}s, \quad \sigma \geqslant 0\right\},$$

$$D_2 = \left\{(\beta, \sigma) \,|\, L_f(0) \leqslant \beta \leqslant L_f(0) + \int_0^\sigma v_r^{\mathrm{II}}(s)\,\mathrm{d}s, \quad \sigma \geqslant 0\right\}.$$

In region D_1, since the initial condition $c^{\mathrm{II}}(\beta, 0) = 0$, and along characteristics $c^{\mathrm{II}}(\beta, \sigma)$ is constant, we have

$$c^{\mathrm{II}}(\beta, \sigma) = 0, \quad (\beta, \sigma) \in D_1.$$

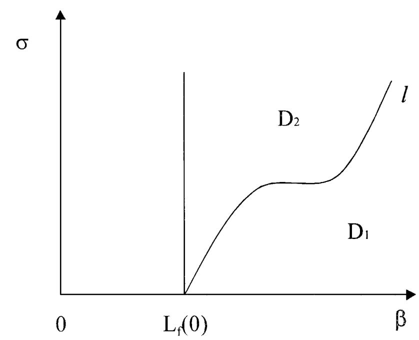

Fig. 1.

In region D_2, the solution is given by Eq. (B14). Therefore, the solution given by authors is only valid in a part of the region D.

It is easy to check that $c^{\mathrm{II}}(\beta, \sigma)$ has a jump discontinuity at curve l. When β approaches l from the left side, we deduce

$$c^{\mathrm{II}}(\beta, \sigma) \,|\, l^- = [v_r^{\mathrm{I}}(0)/v_r^{\mathrm{II}}(0)] f^{\mathrm{I}}(L_f(0))$$

and when β approaches l from the right side, we have

$$c^{\mathrm{II}}(\beta, \sigma) \,|\, l^+ = 0.$$

Thus, $c^{\mathrm{II}}(\beta, \sigma)$ has a jump at the curve l.

The jump discontinuity for $c^{\mathrm{II}}(\beta, \sigma)$ in the region D is an important property, which often arises in the physics associated with such laws as conservation of momentum or conservation of energy.

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Reference

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Reply to comments on "Convection–diffusion of solutes in media with piecewise constant transport properties"

The following is our formal reply to the comment made by Ma and Chang regarding our Chem. Eng. Sci. article (Vaidya *et al*., CES 51, 5299).

"We agree with the observation of Ma and Chang that the complete solution to the system in question must account for the jump discontinuity associated with the characteristic curve. In general, this is indeed an important property of the solution. However, for the systems to which we intended to apply the analysis we expect the effect to be minor. It would appear at the downstream edge of the distribution, where it would cause the concentration to undergo a step change to zero from an already small value."

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